

# WIDEBAND STAP (WB-STAP) FOR PASSIVE SONAR

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## Abstract

*This paper addresses an innovative method for passive sonar signal processing where it is required to suppress a field of moving acoustic interferes while simultaneously enhancing the signal from a weak moving source. Motivated by the Space-Time Adaptive Processing (STAP) technology for similar radar applications, we propose a solution to the passive broadband sonar problem using Wideband Space-Time Adaptive processing (WB-STAP) in the space-frequency domain.*

*The overall objective is to develop models of the source/receiver dynamics directly into the problem formulation and analyze time varying nature of the data vector and the covariance matrix by operating in the spatio-frequency domain. This allows STAP-like formulation for the broadband passive sonar in the space-frequency domain. In particular by employing subaperture techniques and exploiting the underlying model using eigen-subspace methods it is possible to obtain optimum adaptive weight vectors that require significantly less number of data samples compared to conventional methods. In addition, this formulation allows simultaneous tracking of all targets present in the field of view by jointly estimating their arrival angles and doppler parameters.*

## 1. INTRODUCTION

One of the fundamental problems in passive sonar array signal processing is the ability to suppress a

field of moving acoustic interferes while simultaneously enhancing the signal from a desired moving source that may be weaker in strength compared to the interfering signals. The field of view of the passive array may have several moving acoustic sources located along directions  $\{\theta_k\}$  and moving with different velocities  $\{v_k\}$ . Further the acoustic field so generated may be broadband in nature so that the problem reduces to essentially “motion compensation” and coherent integration for passive broadband signals in presence of strong interferers. Moving sources generate equivalent doppler components  $\{\omega_{d_k}\}$  so that the unknown parameter set becomes  $(\theta_k, \omega_{d_k})$ ,  $k = 1, 2, \dots, K$ , where  $K$  represents the total (unknown) number of signals present in the field of view and  $\theta_k$  and  $\omega_{d_k}$  represent the direction of arrival (DOA) and doppler parameter associated with the  $k$ th source.

Similar scenarios exist in radar where a set of pulses interrogate a target that is moving along the direction  $\theta_k$  and extending a doppler velocity  $\omega_d$  at the array [1,2]. In that case Space-Time Adaptive processing (STAP) has been successfully applied to suppress the dominant clutter and interferers and enhance the weak signal that is buried in noise. Adaptive beamforming techniques require the knowledge of the extended space-time covariance matrix, and as a result sample support prob-

Report Documentation Page				Form Approved OMB No. 0704-0188	
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1. REPORT DATE <b>01 SEP 2003</b>		2. REPORT TYPE <b>N/A</b>		3. DATES COVERED <b>-</b>	
4. TITLE AND SUBTITLE <b>Wideband Stap (WB-STAP) For Passive Sonar</b>				5a. CONTRACT NUMBER	
				5b. GRANT NUMBER	
				5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S)				5d. PROJECT NUMBER	
				5e. TASK NUMBER	
				5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) <b>Dept. of Electrical Engg. Polytechnic University Brooklyn, NY 11201</b>				8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)				10. SPONSOR/MONITOR'S ACRONYM(S)	
				11. SPONSOR/MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION/AVAILABILITY STATEMENT <b>Approved for public release, distribution unlimited</b>					
13. SUPPLEMENTARY NOTES <b>See also ADM002146. Oceans 2003 MTS/IEEE Conference, held in San Diego, California on September 22-26, 2003. U.S. Government or Federal Purpose Rights License.</b>					
14. ABSTRACT					
15. SUBJECT TERMS					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT <b>UU</b>	18. NUMBER OF PAGES <b>5</b>	19a. NAME OF RESPONSIBLE PERSON
a. REPORT <b>unclassified</b>	b. ABSTRACT <b>unclassified</b>	c. THIS PAGE <b>unclassified</b>			

lem arises from the nonstationary nature of clutter and interferences present in the data samples that are used to estimate the sample covariance matrix.

To address this problem in the radar context, a variety of subspace based “sample data enhancing techniques” are available [3,4]. To be specific, other available degrees-of-freedom, such as

- (i) subspace based eigencanceller methods
- (ii) subarray-subaperture spatio-temporal smoothing methods
- (iii) convex projection approach and relaxed nonexpanding operators to project the covariance matrix onto the intersection of desired convex sets and
- (iv) optimum transmit-receiver (OTR) waveform design methods maximizing output signal to interference and noise ratio (SINR)

can be incorporated into the adaptive weight vector design to reduce the number of data samples required for its computation. However for radar applications STAP works mainly under the narrowband assumption, and in the broadband sonar context the narrowband methods are not directly applicable. By reexamining the sonar problem in the space-time domain it is possible to extent the STAP strategies to the sonar problem using Wideband Space Time Adaptive processing (WB-STAP).

In what follows we briefly describe the STAP problem, and motivated by that approach a solution to the broadband sonar problem is outlined in the next section using WB-STAP.

## 2. SPACE-TIME ADAPTIVE PROCESSING

Space time adaptive processing (STAP) has emerged as a key enabling technology for airborne intelligence, surveillance and targeting using adaptive MTI radar [1]. With its ability to simultaneously cancel sidelobe jammers and competing doppler-spread clutter returns, STAP has the potential for excellent

detection performance over much of the available angle-doppler spectrum. In normal STAP operation, an estimate of the “target-free” total interference (clutter plus jammer plus thermal noise) covariance matrix must be estimated for a given range-bin under test in order to calculate the optimum set of space-time weight vectors [1,4]. In practice, this estimate is obtained by combining the returns from range-bins surrounding the test cell (with an appropriate number of “guard cells” to prevent target range sidelobe spillover). For a typical surveillance radar, a minimum of several hundred independent and identically distributed (i.i.d.) range-bins would be required if full degree-of-freedom (DOF) STAP was to be employed.

In summary, a major issue in space-time adaptive processing (STAP) for airborne moving target indicator (AMTI) radar both for identification and localization is the sample support problem.

In this context, consider the radar scenario where the returned space-time snapshot signal consists of a target echo and interferences such as jammer, clutter and thermal noise given by

$$\mathbf{x}_i = \alpha_t \mathbf{a}_t + \mathbf{c}_i, \quad (1)$$

where  $\alpha_t$  and  $\mathbf{a}_t \triangleq \mathbf{a}(\theta_t, \omega_{dt})$  are the complex target attenuation factor and target steering vector respectively associated with the spatial and doppler parameters  $\theta_t$  and  $\omega_{dt}$  of the moving target, and  $\mathbf{c}_i$  represents the total interference plus noise signal [1,4]. Here  $\mathbf{x}_i \in C^{MN}$  represents the concatenated space-time data vector formed from the array output vectors corresponding to  $M$  pulse returns that are present in a coherent processing interval (CPI) with interpulse interval equal to  $T$ . Using  $N$  antenna elements, if  $x_k(t_i)$  represents the  $k$ -th sensor output at  $t = t_i$ , then

$$\mathbf{x}(t_i) \triangleq [x_1(t_i), x_2(t_i), \dots, x_N(t_i)]^T \quad (2)$$

represents the array spatial output at time  $t_i$  and

$$\mathbf{x}_i \triangleq [\mathbf{x}(t_i), \mathbf{x}(t_i + T), \dots, \mathbf{x}(t_i + (M-1)T)]^T \quad (3)$$

represents the space-time snapshot data vector due to  $M$  pulses. In the point-doppler estimation problem the optimum weight vector  $\mathbf{w}$  is given by [4]

$$\mathbf{w} = R^{-1} \mathbf{a}_t, \quad (4)$$

where

$$R = E\{\mathbf{c}_i \mathbf{c}_i^*\} \quad (5)$$

is the total interference plus noise covariance matrix. In actual practice, the interference covariance matrix  $R$  must be estimated by making use of the returns from the neighboring range bins to the point of interest. This is most commonly accomplished with the formation of a sample covariance estimate given by [4]

$$\hat{R} = \frac{1}{n} \sum_{j=1}^n \mathbf{x}_j \mathbf{x}_j^* = \frac{1}{n} Y_n Y_n^*, \quad (6)$$

where

$$Y_n \triangleq [\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n]. \quad (7)$$

Unfortunately, for the above procedure to work it is necessary to assume stationarity of the clutter over all range bins used in (6). Moreover since the size of  $R$  is large ( $MN \times MN$ ), (6) requires a very large number of range bins ( $n > MN$ ) to guarantee a good estimate  $\hat{R}$  that is nonsingular, and that much data may not be available in practice.

It has been shown that preprocessing methods – the forward/backward subaperture methods for direct data reduction and convex projection methods for preconditioning sample covariance matrices – that help to significantly reduce required data samples in estimating the sample covariance matrix can be employed on the limited data set for superior performance [3,4].

Motivated by the STAP methodology, next we address the adaptive processing for wideband passive sonar signals.

### 3. SPACE-FREQUENCY ADAPTIVE PROCESSING

Consider a set of broadband signals  $s_1(t), s_2(t), \dots, s_K(t)$  that are located along directions  $\theta_1, \theta_2, \dots, \theta_K$  with respect to an array, and moving with effective velocities  $v_1, v_2, \dots, v_K$  (along the wavefront). A set of  $N$  passive sensors is used to collect the data arriving from these sources. The sensors can be arranged in various configurations (linear, circular, rectangular towed arrays etc.), however, for the present discussion we will assume a uniformly spaced linear array with interelement spacing  $d$ . Assume that the source located along direction  $\theta$  is moving along that direction with an effective velocity  $v$  away from the array. At  $t = t_0$ , let the target be at a distance  $r$  along that direction with respect to the first sensor. Then if  $s(t_0)$  represents the target waveform, then the first sensor output  $x_1(t)$  at that instant is given by

$$x_1(t_0) = s(t_0 - r/c) \quad (8)$$

and since the ray has to travel an additional distance of  $d \sin \theta$  to arrive at the second sensor we get the second sensor output to be

$$x_2(t_0) = s[t_0 - (r + d \sin \theta)/c], \quad (9)$$

where  $c$  represents the uniform underwater acoustic velocity. Notice that (9) contains information only about the direction of arrival  $\theta$ . To estimate the velocity and the associated doppler parameter, consider the array output at time  $t = t_0 + T$ . The target moves a distance  $vT$  along the ray and hence the first sensor output at  $t = t_0 + T$  is given by

$$x_1(t_0 + T) = s[t_0 + T - (r + vT)/c]$$

$$\begin{aligned}
&= s[t_0 - \frac{r}{c} + T(1 - v/c)] \\
&= s[t_0 - \frac{r}{c} + T\omega_d], \quad (10)
\end{aligned}$$

where we define

$$\omega_d \triangleq 1 - \frac{v}{c} \quad (11)$$

to be the effective doppler velocity associated with the moving target. From (9) the second sensor output at  $t = t_0 + T$  is given by

$$x_2(t_0 + T) = s[t_0 - \frac{r}{c} + T\omega_d - d \sin \theta/c]. \quad (12)$$

Equations (8)-(12) can be used to determine the signal strength arriving from  $K$  moving sources located along directions  $\theta_1, \theta_2, \dots, \theta_K$ , and moving with velocities  $v_1, v_2, \dots, v_K$  away from the array. From (8)-(9), the  $i$ th sensor output is given by

$$x_i(t) = \sum_{k=1}^K s_k(t - (i-1)\frac{d}{c} \sin \theta) + w_i(t), \quad (13)$$

where  $s_k(t)$  represents the reference signal of the  $k$ th source at the first sensor. Similarly using (8)-(12), we obtain the output at time  $t + nT$  to be

$$\begin{aligned}
x_i(t + nT) &= \sum_{k=1}^K s_k(t - (i-1)\frac{d}{c} \sin \theta_k \\
&+ nT\omega_d) + w_{i,n}(t), \quad (14) \\
&i = 1, 2, \dots, M, \quad n = 1, 2, \dots, N.
\end{aligned}$$

Here  $w_{i,n}(t)$  represents the noise and other reverberation components present in the scene.

Eqs (13)-(15) represents the data available, and to make further progress it is useful to represent this set of data in the frequency domain.

Let  $X_i(\omega, n)$  represent the Fourier transform of  $x_i(t + nT)$  in (15). This gives

$$\begin{aligned}
X_i(\omega, n) &= \sum_{k=1}^K S_k(\omega) e^{-j(i-1)\frac{d}{c} \sin \theta_k} e^{jnT\omega_d} \\
&+ W_{i,n}(\omega), \quad (15)
\end{aligned}$$

where  $S_k(\omega)$  represents the Fourier transform of the  $k$ th source output  $s_k(t)$ . Notice that Eq. (15) represents the available space-frequency data.

Then (15) can be expressed compactly as

$$\mathbf{X}(\omega) = \sum_{k=1}^K S_k(\omega) \mathbf{A}(\omega, \theta_k, \omega_{d_k}) + W(\omega). \quad (16)$$

Following STAP formulation in Sec. 2, let

$$\mathbf{R}(\omega) = E [\mathbf{X}(\omega) \mathbf{X}^*(\omega)] \quad (17)$$

represents the  $MN \times MN$  covariance matrix in the space-frequency domain. Then for a desired target located along  $(\theta_0, \omega_{d_0})$ , we get the optimum weight vector at frequency  $\omega$  to be [as in (4).]

$$\mathbf{W}(\omega) = R^{-1}(\omega) \mathbf{A}(\omega, \theta_0, \omega_{d_0}). \quad (18)$$

Notice that direct inversion of (17) requires at least  $MN$  samples for estimating (17), and in a nonstationary scene it is necessary to examine alternate procedures to estimate the adaptive weight vector. In this context, if the signal scene consists of  $K$  sources, then an eigendecomposition of  $\mathbf{R}(\omega)$ , as well as the adaptive weight vector in (18) can be expressed with  $r$  replaced by  $K$  there.

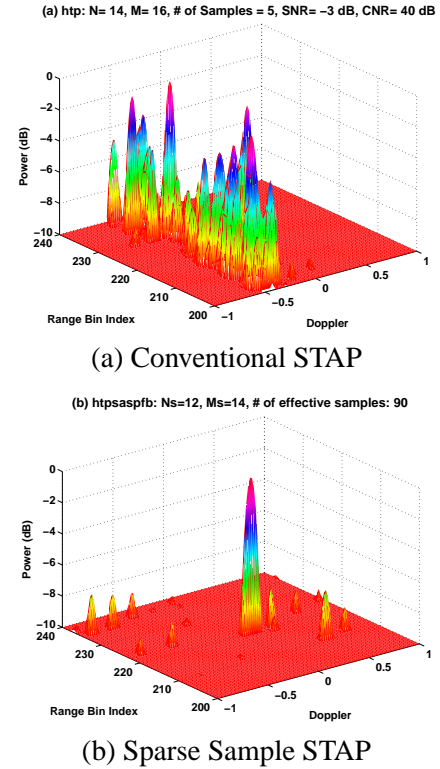
For example, a 14 sensor array with 16 pulses used in Fig.1a results in a covariance matrix of dimension  $14 \times 16 = 224$ , and traditionally atleast double the number of samples are required for 3dB performance [4]. In Fig.1b, the signal is buried in 40 dB clutter to noise ratio (CNR) with  $-3$  dB SNR and it is detected with an SINR improvement factor of 8 dB using five samples – compared to  $2 \times 224 = 448$  samples. Fig. 2 shows the extension of these ideas to joint Doppler-frequency and angle of arrival estimation for the broadband passive sonar problem in the space-frequency domain for a five source scene with 10 dB SNR and using six sensors. The data structure in (16)-(17) is exploited here by utilizing the eigensubspace methods in the frequency domain.

#### 4. CONCLUSIONS

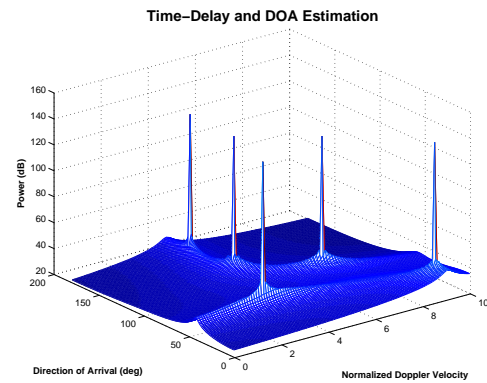
The overall objective is to develop models of the source/receiver dynamics directly into the problem formulation and analyze time varying nature of the data vector and the covariance matrix by operating in the spatio-frequency domain. This allows STAP-like formulation for the broadband passive sonar in the space-frequency domain (WB-STAP). In particular by employing subaperture techniques and exploiting the underlying model using eigen-subspace methods it is possible to obtain optimum adaptive weight vectors that require significantly less number of data samples compared to conventional methods. In addition, this formulation allows simultaneous tracking of all targets present in the field of view by jointly estimating their arrival angles and doppler parameters.

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**Fig. 1.** Adaptive matched filter output responses of (a) conventional STAP and (b) sparse sample STAP methods using five snapshot data samples. A fourteen element array with sixteen pulses receives data with  $\text{CNR} = 40\text{dB}$ . Injected target with  $\text{SNR} = -3 \text{ dB}$  is located at range bin 220.



**Fig. 2.** Joint doppler-frequency and angle of arrival estimation of arrivals using WB-STAP. The sources are located at 42, 151, 108, 73 and 11 degrees with normalized doppler velocities of 2.8, 3.9, 4.1, 6.5 and 8.9.